Angles, Azimuths, and Bearings
Terms:

- **Meridian**: A line on the mean surface of the earth joining the north and south poles.
- **Geographic North**: True North Sometimes called Geodedic True North-fixed
- **Magnetic North**: Taken from a magnetic compass-changes with time
- **Grid North (Meridian)**: Lines parallel to a grid reference meridian (central meridian)-SPC
Terms:

- **Bearing** - The direction of a line as given by the acute angle between the line and a reference meridian.

- **Azimuth** – The direction of a line as given by an angle measurement clockwise (usually) from the north end of a reference meridian. *(NAD 83 is from the North) (NAD 27 is from the South)*
Interior, Exterior, and Deflection Angles

- For all closed polygons of \( n \) sides, the sum of the interior angles will be \( (n-2)180^\circ \), and the sum of the exterior angles will be \( (n+2)180^\circ \).
  - Interior
  - Deflection Angles
    - Roadway Centerline Alignments
    - Horizontal Curves
Figure 7.2  Closed polygon. (a) Clockwise interior angles (angles to the right). (b) Counter-clockwise interior angles (angles to the left).
Interior, Exterior, and Deflection Angles

Figure 7.3  Deflection angles.
Figure 7.4  Azimuths.
Figure 7.5  Bearing angles.
<table>
<thead>
<tr>
<th>Azimuths</th>
<th>Bearings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vary from 0 to 360°</td>
<td>Vary from 0 to 90°</td>
</tr>
<tr>
<td>Require only a numerical value</td>
<td>Require two letters and a numerical value</td>
</tr>
<tr>
<td>May be geodetic, astronomic, magnetic,</td>
<td>Same as azimuths</td>
</tr>
<tr>
<td>grid, assumed, forward or back</td>
<td></td>
</tr>
<tr>
<td>Are measured clockwise only</td>
<td>Are measured clockwise and counterclockwise</td>
</tr>
<tr>
<td>Are measured either from north only, or</td>
<td>Are measured from north and south</td>
</tr>
<tr>
<td>from south only on a particular survey</td>
<td></td>
</tr>
</tbody>
</table>
Bearing – Azimuth Conversions

Convert from bearing to azimuths

- Convert from bearing to azimuths by using these relationships:
  - 1. NE quadrant: azimuth = bearing
  - 2. SE quadrant: azimuth = 180° – bearing
  - 3. SW quadrant: azimuth = 180° + bearing
  - 4. NW quadrant: azimuth = 360° - bearing

Reverse Directions

- Back azimuth (reverse direction) = azimuth + / - 180°
- Back bearing (reverse direction) = same numeric value with opposite directions
A reverse bearing is a bearing in the opposite direction. For example a bearing in the NW quadrant becomes a bearing of the same angle in the SE quadrant.
Example 1

Find these directions:

<table>
<thead>
<tr>
<th></th>
<th>Azimuth</th>
<th>Bearing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line 0-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Line 0-2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Line 0-3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Line 4-0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Solution

Find these directions:

<table>
<thead>
<tr>
<th>Line</th>
<th>Azimuth</th>
<th>Bearing</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-1</td>
<td>58° 41' 38''</td>
<td>N 58° 41' 38'' E</td>
</tr>
<tr>
<td>0-2</td>
<td>140° 11' 32''</td>
<td>S 33° 48' 28'' E</td>
</tr>
<tr>
<td>0-3</td>
<td>214° 28' 29''</td>
<td>S 34° 20' 39'' W</td>
</tr>
<tr>
<td>0-0</td>
<td>241° 45' 15''</td>
<td>N 18° 14' 45'' W</td>
</tr>
</tbody>
</table>

\[
\text{Line 0-2} \Rightarrow A_2 = 58° 41' 38'' + 87° 29' 54'' = 146° 11' 32'' \\
\text{Bearing} = 180° - 146° 11' 32'' = S 33° 48' 28'' E
\]

\[
\text{Line 0-3} \Rightarrow A_2 = 270° - 55° 31' 21'' = 214° 28' 39'' \\
\text{Bearing} = 214° 28' 39'' - 180° = S 34° 20' 39'' W
\]

\[
\text{Line 0-4} \Rightarrow A_2 = 360° - 18° 14' 45'' = 341° 45' 15'' \\
\text{Bearing} = N 18° 14' 45'' W
\]
Example 2

Compute the interior angles of this closed traverse.
Solution

Compute the interior angles of this closed traverse.

\[ \angle J = 180' - 51'52''50'' - 13'23'53'' = 108'43'57'' \]

\[ \angle K = 180' - 55'50''50'' = 70'49'40'' \]

\[ \angle L = 180' - 15'52'10'' = 16'08'40'' \]

\[ \angle G = 78'41'24'' + 16'08'40'' = 94'50'04'' \]

\[ \angle H = 101'18'36'' + 13'28'53'' = 114'42'09'' \]

CHECK: \( n - 2 \) \( 180 = 540^\circ00'00'' \)
Traverse loop azimuth computations

1. To compute azimuths in the counterclockwise direction, add the interior angle to the back azimuth of the previous course.

2. To compute azimuths in the clockwise direction, subtract the interior angle from the back azimuth of the previous course.

See the Traverse Example Problem for an example of this computation.

Azimuth computations Practice Problem

Find the azimuth of each leg, proceeding in the counterclockwise direction.
Azimuth computations Practice Problem

Find the azimuth of each leg, proceeding in the counterclockwise direction.

\[
\begin{align*}
A_{E-Q-R} &= 49 \degree 18' 19'' \\
&+ 92 \degree 58' 01'' \\
&= 142 \degree 16' 20''
\end{align*}
\]

\[
\begin{align*}
A_{E-P-O} &= 286 \degree 22' 32'' \\
&- 180 \degree 00' 00'' \\
&+ 122 \degree 55' 47'' \\
&= 229 \degree 18' 19''
\end{align*}
\]

\[
\begin{align*}
A_{S-T} &= 257 \degree 1' 41'' \\
&+ 92 \degree 46' 04'' \\
&= 349 \degree 47' 45''
\end{align*}
\]

\[
\begin{align*}
A_{T-P} &= 169 \degree 47' 45'' \\
&+ 11 \degree 34' 47'' \\
&= 286 \degree 22' 32'' \\
\text{CHECKS}
\end{align*}
\]
Magnetic Declination

http://www.ngdc.noaa.gov/geomagmodels/Declination.jsp

Knoxville, TN 2012: 5°23’W 0°4’W per yr.
Figure 7.10  Isogonic lines from World Magnetic Model for 2005. This image is from the NOAA National Geophysical Data Center, NGDC on the Internet at http://www.ngdc.noaa.gov/seg/geomag/declination.shtml

US/UK World Magnetic Model – Epoch 2005.0
Main Field Declination (D)
<table>
<thead>
<tr>
<th>City</th>
<th>Magnetic Declination</th>
<th>Annual Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boston, MA</td>
<td>15°48′W</td>
<td>1.0′E</td>
</tr>
<tr>
<td>Cleveland, OH</td>
<td>7°45′W</td>
<td>4.1′W</td>
</tr>
<tr>
<td>Madison, WI</td>
<td>1°13′W</td>
<td>6.2′W</td>
</tr>
<tr>
<td>Denver, CO</td>
<td>10°21′E</td>
<td>4.9′W</td>
</tr>
<tr>
<td>San Francisco, CA</td>
<td>15°26′E</td>
<td>3.1′W</td>
</tr>
<tr>
<td>Seattle, WA</td>
<td>18°45′E</td>
<td>7.2′W</td>
</tr>
</tbody>
</table>

Table 7.4  MAGNETIC DECLINATION AND ANNUAL CHANGE FOR VARIOUS LOCATIONS IN THE UNITED STATES FOR JANUARY 1, 2000
Magnetic Declination
http://www.ngdc.noaa.gov/geomagmodels/Declination.jsp

8. Magnetic Declinations

A. Method 1 - Magnetic Declinations by Table

Magnetic declination is the amount of change that Magnetic North differs from True Geodetic North. This difference is measured east or west of True Geodetic North. Navigators call this change variation, while the military call it deviation. A good way to remember this is when going from True to Magnetic - “east is least and west is best.” That is, always subtract the east declination and add the west declination.

- E(+), W(-)
  - Old magnetic bearing
  - Old magnetic azimuth
  - Old magnetic declination E(+), W(-)
  - True azimuth
- E(-), W(+)
  - Present magnetic declination E(-), W(+)
  - Present magnetic azimuth
  - Present magnetic bearing
  - True bearing
Example 1:
The true bearing of a line is S 87°30' W. What is the magnetic bearing of this line if the declination is 8°15' W?

<table>
<thead>
<tr>
<th>E(-)</th>
<th>T</th>
<th>Magnetic declination = + 8°15'</th>
</tr>
</thead>
<tbody>
<tr>
<td>W(+)</td>
<td>M</td>
<td>Magnetic bearing = N 84°15' W</td>
</tr>
<tr>
<td></td>
<td></td>
<td>True azimuth = 267°30'</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Magnetic azimuth = 275°45'</td>
</tr>
</tbody>
</table>

Example 2:
If the magnetic bearing of a line is N 06°15' W at a certain location when the declination is 7°30' E, what is the true bearing?

<table>
<thead>
<tr>
<th>E(+)</th>
<th>M</th>
<th>Magnetic bearing = N 06°15' W</th>
</tr>
</thead>
<tbody>
<tr>
<td>W(-)</td>
<td>T</td>
<td>Magnetic Declination = + 7°30'</td>
</tr>
<tr>
<td></td>
<td></td>
<td>True azimuth = 1°15'</td>
</tr>
<tr>
<td></td>
<td></td>
<td>True bearing = N 01°15' E</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Magnetic azimuth = 353°45'</td>
</tr>
</tbody>
</table>

Example 3:
The magnetic declination of an old line was 2°00' W when the magnetic bearing S 85°30' E. The present magnetic declination in the area is 5°30' W. What is the present magnetic bearing of the line?

<table>
<thead>
<tr>
<th>E(+)</th>
<th>M</th>
<th>Old magnetic bearing = S 85°30' E</th>
</tr>
</thead>
<tbody>
<tr>
<td>W(-)</td>
<td>T</td>
<td>Old magnetic azimuth = 94°30'</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Old magnetic declination = - 2°00'</td>
</tr>
<tr>
<td></td>
<td></td>
<td>True azimuth = 92°30'</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Present magnetic declination = + 5°30'</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Present magnetic azimuth = 98°00'</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Present magnetic bearing = S 82°00' E</td>
</tr>
<tr>
<td></td>
<td></td>
<td>True azimuth = 92°30'</td>
</tr>
</tbody>
</table>
B. Method 2 — Magnetic Declination by Sketch

Drawing a simple sketch may also solve problems involving magnetic declination. When doing this, it is usually easier to work with azimuths. Consider the following example, which is the same as exercise 5.2:

1910 magnetic bearing is N 25° 30' E
1910 declination is 3° 30' W
Find the present magnetic bearing if the current declination is 5° W

SOLUTION:

STEP 1 Find the true azimuth in 1910 by sketch.

In the above sketch, the true azimuth is obviously 25° 30' - 3° 30', which is 22°. Since there is no annual change for True North, the present true azimuth is also 22°.

STEP 2 Find the current magnetic bearing by a new sketch.

The current magnetic azimuth is obviously 22° + 5° = 27°, and the magnetic bearing is N 27° E.
Example 7.3
Assume the magnetic bearing of a property line was recorded as S43-30E in 1862. At that time the magnetic declination at the survey location was 3-15W. What geodetic bearing is needed for a subdivision property plan?

Solution:

Geodetic Bearing = S43-30E + 3-15 = S46–45E

Figure 7.12  Computing geodetic bearings from magnetic bearings and declinations.
Example 5.1

Given: Mag Bearing = N 10°30'E  
      Declination of 15°30'E

Det: Mag Bearing 6 in Sept 2001

From Chart: Declination in 2003 = 18°50'E
            Annual Change = 8' W/yr

1st Find True Bearing — Remember True North! True Bearings Neuer
Change, Only the Mag North

\[ M - T \ E(\theta) \ W(\lambda) \]
* From Jan 2000 to Sept 2001 = 1.75 yrs

* The change in declination would be \(1.75 \times 8\, \text{W/yr} = 14\, \text{W}\)

  Therefore, the declination in Sept 2001 is 14\, \text{W} of the 18\, \text{SDE} declination in 2000

\[\Rightarrow \text{Sept 2001 Declination} = 18\, \text{SDE} - 0\, 14\, \text{W} = 18\, \text{SDE}\]

* Find Mag Bearings For Sept 2001

\[\text{N24°00'E - 18°36'E} = \overline{N7°24'E}\]
Given: Mag. Bearing Jan 2, 2000 N39°30'E

Def: a) Mag. Bearing Jan 2, 2004
    b) Astronomic (true) Bearing

Use 19°E as Declination in 2000 & Rate of Change 8'N/yr

1st Find True Bearing
• From 1/2/2000 to 7/2/2004 = 4.5 yrs
  \[ (4.5\text{ yr}) \times 8\text{ W/yr} = 36\text{ W} \]

• Mag. North has moved 0° 36' W since 1/2/2000. Therefore, making the declination in 7/2/2004 \( 19° - 0° 36' W = 18° 24' E \)

• Find Mag. Bearing for 7/2/2004

\[
\begin{array}{c|c|c}
\text{UB° 30'E} & \text{True} & \text{Declination} \\
\hline
\ 18° 24' E & 18° 24' E \ & \ N 40° 06'E \ & \text{Mag} \\
\end{array}
\]